Factorization of Polynomials

Name:____________________( )

Class: F. 3( ) Date: ______________

Important Terms

Factorization	因式分解	Identity	恆等式
Common Factor	公因式	Grouping Terms	併項
Polynomial	多項式	Expansion	展開

Revision Notes:

1. More about Factorization of Polynomials

(a) $5x$ and $2x^2 + 1$ are the factors of the polynomial $10x^3 + 5x$. ◀ $10x^3 + 5x = 5x(2x^2 + 1)$

$5x$ is the common factor of all the terms of $10x^3 + 5x$.

(b) To factorize a polynomial is to express the polynomial as a product of its factors.

Example 1. $10x^3 + 5x = 5x(2x^2 + 1)$

(c) Factorization of polynomials is the reverse process of expansion.

(d) There are different methods of factorization. The commonly used methods are: taking out common factors, grouping terms, using identities and the cross method.

2. Taking out Factors and Grouping Terms

(a) Taking out common factors

Example 2. $-5a - 5b = -5(a + b)$

(b) Grouping terms

Example 3. $cd - c + d^2 - d = c(d - 1) + d(d - 1)$

$= (d - 1)(c + d)$

3. Using Identities

(a) The following identities can be used to factorize the polynomials in the form of $a^2 - b^2$, $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$.

$$a^2 - b^2 \equiv (a + b)(a - b)$$
$$a^2 + 2ab + b^2 \equiv (a + b)^2$$
$$a^2 - 2ab + b^2 \equiv (a - b)^2$$

Example 4. Factorize (i) $9x^2 - 16y^2$.

(ii) $16a^4 - 36$

(iii) $(4 + ab^2)^2 - 4a^2b^4$

Solution (i) $9x^2 - 16y^2 = (3x)^2 - (4y)^2$

$= (3x + 4y)(3x - 4y)$

(ii) $16a^4 - 36 = 4(4a^4 - 9)$

$= 4[(2a^2)^2 - (3)^2]$

$= 4(2a^2 + 3)(2a^2 - 3)$

(iii) $(4 + ab^2)^2 - 4a^2b^4 = (4 + ab^2)^2 - (2ab^2)^2$

$= (4 + ab^2 + 2ab^2)(4 + ab^2 - 2ab^2)$

$= (4 + 3ab^2)(4 - ab^2)$

Example 5. Factorize(i) $x^2 + 8x + 16$

(ii) $25x^2 - 30xy + 9y^2$

Solution (i) $x^2 + 8x + 16 = (x)^2 + 2(4)(x) + (4)^2$

$= (x + 4)^2$

(ii) $25x^2 - 30xy + 9y^2 = (5x)^2 - 2(5x)(3y) + (3y)^2$

$= (5x - 3y)^2$

(b) The following identities can be used to factorize the polynomials in the form of $a^3 - b^3$ or $a^3 + b^3$:

$$a^3 - b^3 \equiv (a - b)(a^2 + ab + b^2)$$
$$a^3 + b^3 \equiv (a + b)(a^2 - ab + b^2)$$

Example 6. Factorize $135 - 40a^3$.

Solution $135 - 40a^3 = 5(27 - 8a^3)$

$= 5[(3)^3 - (2a)^3]$

$= 5(3 - 2a)[(3)^2 + (3)(2a) + (2a)^2]$

$= 5(3 - 2a)(9 + 6a + 4a^2)$

4. Cross method

This method can be used to factorize quadratic polynomials of the form $px^2 + qx + r$.

e.g. Factorize $x^2 - 4x + 3$.

Since the x^2 term can be written as $(x) \cdot (x)$, and the constant term + 3 can be written as (+1) (+3) or (–1) (–3), we can list out all the possible pairs of factors as follows:

x	+1	–1
x	+3	–3

Using the cross method to test each possible pair of factors, we have $x^2 - 4x + 3 = (x - 1)(x - 3)$